EXPRESSIONS AND EQUATIONS 2 STUDENT PACKET

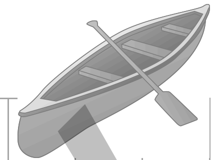
EXPRESSIONS AND BALANCE

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 2.5.) Key mathematical vocabulary is underlined throughout the packet.

consecutive integers	distributive property
equation	expression
equivalent expressions	like terms
variable	word of your choice: __________________

THE LAKE PROBLEM



Follow your teacher's directions to explore this problem.

THE LAKE PROBLEM
(Continued)

3. Review your work and notes. Do you see any patterns? Does anything seem to be happening regularly, over and over again? Circle a repeating pattern if you see one. Write your observations below.

4. Write a numerical expression that represents the number of trips it takes for 6 adults and 2 children to cross the lake.

_____ • _____ + _____

For problems 5-10, write each as an expression in the form of problem 4 above. Use your diagram as needed to determine the number of one-way trips necessary to get each combination of people across the lake.

5. 4 adults and 2 children	6. 2 adults and 2 children
7. 0 adults and 2 children	8. 20 adults and 2 children
9. 100 adults and 2 children	10. x adults and 2 children

11. Explain the meaning of the expression in problem 10 above?

12. Suppose it takes some adults and 2 children a minimum of 201 one-way trips to get everyone across the lake. How many adults are in the group?

EQUIVALENT EXPRESSIONS: INTRODUCTION TO CUPS AND COUNTERS

We will apply properties of arithmetic to generate equivalent expressions that include integers. We will write numerical expressions to represent geometric patterns, describe patterns in words, and generalize these patterns using variable expressions.

GETTING STARTED

1. Find variable and expression in section 2.5 and explain what they mean in My Word Bank.

In algebra, we typically write expressions horizontally or as fractions. Rewrite each arithmetic problem below as an expression, horizontally on one line. Do not compute.

2. $\begin{array}{r} 23 \\ 46 \\ +\ 54 \\ \hline \end{array}$ ____________	3. $\begin{array}{r} 132 \\ -\ 67 \\ \hline \end{array}$ ____________	4. $19\overline{)451}$ ____________

5. Rewrite problem 4 using fraction notation.

Write an expression for each verbal statement in problems 6 and 7. Do not compute.

6. a. There are 6 puppies and 8 kittens. Write a numerical expression for the number of puppies and kittens.

   ____

   b. There are p puppies and k kittens. Write a variable expression for the number of puppies and kittens.

   ____

7. a. Shira has 4 ribbons. Raza has 6 times as many ribbons as Shira. Write a numerical expression for the number of Raza's ribbons.

   ____

   b. Shira has n ribbons. Raza has 6 times as many ribbons as Shira. Write a variable expression for the number of Raza's ribbons.

   ____

THE DISTRIBUTIVE PROPERTY

This table shows the numbers of hours Hua worked at the Pizza Shop.

Mon	Tue	Wed	Thu	Fri	Sat	Sun
off	4	off	off	4	6	6

1. Circle all expressions below that could describe the number of hours she worked that week.

   A. $4 + 4 + 6 + 6$ B. $2 \cdot 4 + 2 \cdot 6$ C. $2(4 + 6)$

   D. $(4 + 6) + (4 + 6)$ E. $2 \cdot 4 + 6$ F. $(4 + 6)2$

2. The distributive property illustrates equivalent expressions where one is written as a product and the other written as a sum. Look these words up in section 2.5 and explain what they mean in My Word Bank.

3. Choose two equivalent expressions from problem 1 that illustrate the distributive property.

   _______________ = _______________

4. Use the distributive property to rewrite each expression. Then find the value for each to verify that they are equivalent.

	expression written as a product	value of the expression as a product	expression written as a sum	value of the expression as a sum
a.	5(2 + 7)		5(__) + 5(__)	
b.	(__ + __)5		6(5) + 2(5)	
c.	4(7 – 5)			
d.			5(4) – 5(3)	
e.	(6 + 1)9			

5. At Papa's Pizza, the variable expression $p + m$ represents the cost for one order of a slice of pepperoni pizza and a medium drink. Write expressions for the cost of two orders.

   a. As a product: __________ b. As a sum: __________

   c. Papa's Pizza charges $3.00 for a slice of pepperoni pizza and $2.00 for a medium drink. Substitute these values into both expressions above to verify they are equivalent.

INTRODUCTION TO CUPS AND COUNTERS

Follow your teacher's directions to complete problems 1-8 on this page.

(1)

Positive Counter	Negative Counter	Cup

(2)	(3)	(4)
(5)	(6)	(7)
8.		

Build and draw the following expressions.

9. $2x + 1$	10. $x - 4$	11. $2x - 4$

Write variable expressions for the following.

12. **V + + +**	13. **V V**	14. **V V – –**

15. In problem 8 above, you showed that $3(x + 2)$ is equivalent to $3x + 6$. Verify that these expressions have the same value:

a. when $x = 4$:

b. when $x = -4$:

EQUIVALENT EXPRESSIONS

Follow your teacher's directions to complete problems 1-8 on this page.

(1)

(2) Find like terms in section 2.5 and explain what it means in My Word Bank.

(3)	(4)
(5)	(6)

7.	8.

Build and draw the following expressions. Then write each in its simplest form.

9. $2x + x - 3$	10. $2x + x - 5 - 2x$	11. $2(x + 1) + 3x$

Simplify each expression.

12. $12x + 13x + 10$	13. $15x + x - 5 - 4x + 25$	14. $14(x + 1) - 11x$

PRACTICE 1

1. Use the distributive property to rewrite each expression. Then find the value for each to verify that they are equivalent.

	Expression written as a product	Value of the expression as a product	Expression written as a sum	Value of the expression as a sum
a.	4(5 + 7)			
b.			6(3) + 6(7)	
c.	5(8 – 3)			
d.			2(12) – 2(5)	
e.	(7 + 1)9			

2. Consider the algebraic expression: $x + 3x + 5 + 3 + 5x$
   a. Evaluate this expression when $x = 2$.

   b. Simplify the expression by combining like terms.

   c. Evaluate the simplified expression in part b when $x = 2$ to verify that it is equivalent to the original expression.

Simplify each expression.

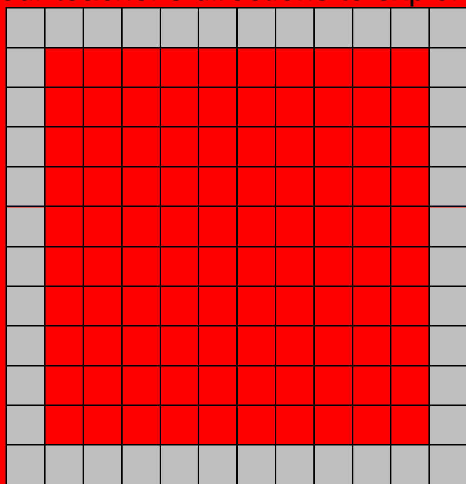
3. $x - 11 + 19x - 9$	4. $18 + 8x + 6(x + 2)$	5. $7(x - 5) + 3x - 5$

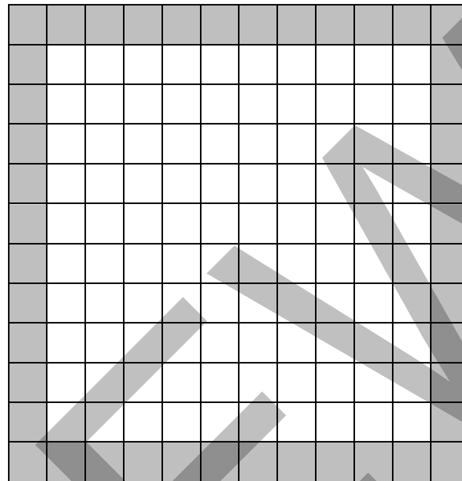
6. Trista looked at the expression $n + 1 + n + 1$ and said, "That's just two groups of $n + 1$, so I'll write it as $2(n + 1)$." Critique Trista's reasoning.

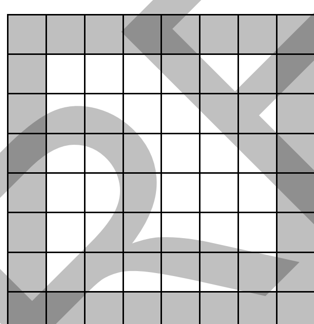
7. Tere looked at the expressions $2n$ and n^2. She substituted the value of 2 for n in both expressions, and then said, "They're both equal to 4, so they must be equivalent expressions." Critique Tere's reasoning.

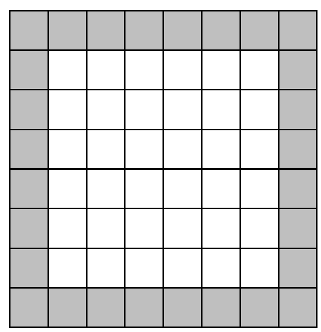
THE BORDER PROBLEM

Follow your teacher’s directions to explore this problem.

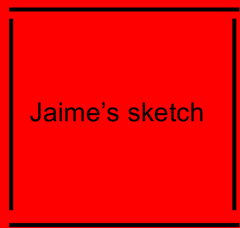
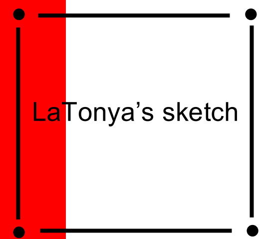
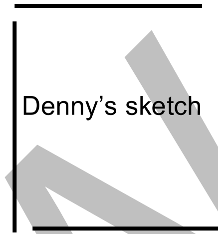








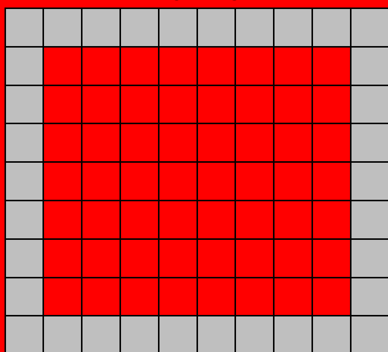
BORDER PROBLEM CHART

	 Jaime's sketch	 LaTonya's sketch	 Denny's sketch
12×12			
8×8			
5×5			
$n \times n$			

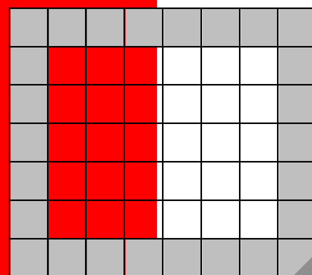
PRACTICE 2

1. Draw sketches and write at least two numerical expressions for the number of shaded border squares in these gardens. Two of the gardens have been drawn for you.

10×9



8×7



5×4

2. Consider the border pattern that seems to be established in the gardens above.

   If the shorter side has length *n*, then the longer side has length (_____ + _____).

   Write at least two variable expressions for the number of shaded border squares and simplify them. Underline the simplified expressions to show that they are equivalent.

SOLVING EQUATIONS: BALANCE

We will simplify expressions within equations. We will solve equations using balance techniques. We will formalize balance techniques with properties of equality.

GETTING STARTED

Compute.

1. $-4 + 5 - 9$	2. $3(2 - 4)$	3. $\frac{-24}{6}$
4. $-12 + 10(2 - 1)$	5. $-(5 - 3)$	6. $5(-2) - 3(2)$

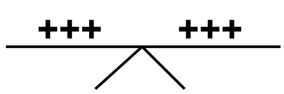
Simplify each expression. Build and draw as needed.

7. $x + 1 + 2x - 3$	8. $4(x - 5) - 2x$	9. $-3 + 4(x + 2)$

Solve each equation using substitution and check.

10. $4 + x = 100$	11. $38 = 5x + 13$	12. $3(x - 8) = 36$

13. Here is a balanced scale. What happens if four counters are added to the left side?



14. Try to solve this equation:

$$3x + 2(x - 2) + 5x - 1 = 0$$

Why is this more challenging to solve than the equations in 10-12 above?

EQUATION SOLVING STRATEGY: BALANCE

Follow your teacher's directions to learn a new equation solving strategy.

V represents ___________ **+** represents _____ **–** represents ______

Write the equation and build it. • Continue the building process until the equation is solved. • Write some steps and the solution. • Check it using substitution.	**Formal algebra steps** **Notes**
(1)	
(2)	
(3)	

PRACTICE 3

Solve each equation using a balance strategy. Build or draw if needed. Check by substitution.

1. $2x + 5 = 13$	2. $-15 = 3(x + 3)$	3. $2x + 1 + 6x = -23$
4. $3x + 8 + 2x = -17$	5. $18 = 3m + 5$	6. $2(4x – 6) = -12$
7. $-3 = -9 + 7x + 2x$	8. $3(2 + x) = -4 + 4$	9. $-21 = 4x + 6x – 1$

VICKI, NICKI, AND RICKI

Vicki and Nicki discussed how they might solve the equation $20d + 78 = 12$.

1. Vicki said, “First I’m going to divide the expressions on both sides of the equation by 20.” Even though Vicki’s strategy is permissible, why might it be difficult to execute?

2. Nicki said, “First I’m going to subtract 78.” Even though Nicki has the right idea, explain why this language is not precise.

3. Solve the equation above. Show your work.

Ricki has \$240 in her savings account. She deposits \$20 per month for several months.

4. Write a numerical expression for the amount of money that is in Ricki’s account after 6 months.	5. Write a variable expression for the amount of money that is in Ricki’s account after *n* months.
6. Write an equation to represent that Ricki’s has \$580 in her account after *n* months. Then solve the equation for *n*.	7. Ricki decides to be more ambitious about saving. With the same initial amount of money in her account and with deposits of \$25 per month, how long will it take for her to save \$580? Show all work by writing and solving an equation.

PRACTICE 4

Solve each equation using a substitution strategy, a balance strategy, or a combination of both. Show your work. Check by substitution.

1. $120 = 3x + 75$	2. $150 = 5(x + 20)$
3. $4 + 7x + 2(8 + 4x) = 170$	4. $1 + 3(x + 5) + 7x = 111$
5. $y + 4y + 23 = 78$	6. $4(p - 6) + 6 = 120 - 10$
7. $-44 = 5(x + 2) + x$	8. $\frac{m}{4} - 9 = 4 - 5$

GEOMETRY REVIEW

Use the definitions in section 2.5 to refresh your memory about geometry.

1. An equilateral triangle has a perimeter of 15 inches.
   Find the length of each side.
   Make a sketch and label the side lengths.

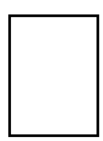
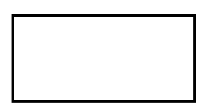
2. An isosceles triangle has two congruent sides that are 5 inches each and the third side that is 9 inches. Make a sketch, label the side lengths, and find the perimeter.

3. The perimeter of a rectangle is 54 cm. Its length is 16 cm. What is its width? Make a sketch to help you solve the problem.

4. The perimeter of a rectangle is $P = L + W + L + W$. Use the distributive property to rewrite this formula in two ways.

   a. As a product: P = ___________ b. As a sum: P = ___________

Use the formulas for perimeter of a rectangle written as a product and written as a sum to find the perimeter of each rectangle in two different ways.

5. 	6. 	7. 

GEOMETRY PROBLEMS

Follow this outline to organize your work when solving problems using algebra.

- Make a sketch and identify the variable(s).
- Write an equation.
- Solve the equation.
- Answer the question.
- Check your answer in the ORIGINAL problem.

1. The perimeter of a rectangular field is 256 feet. The longer sides are each 28 feet more than the shorter sides. Find the dimensions of the field.

2. The perimeter of a rectangular swimming pool is 126 feet. The length is twice the width. Find the length of the pool.

3. In an isosceles triangle, the length of the base is 6 inches less than twice the length of each leg. The perimeter is 30 inches. Find the lengths of the legs and the base.

4. The length of the second side of a triangle is twice the length of the first side. The length of the third side is 4 units more than the first side. The perimeter is 52 units. How long is each side?

NUMBER PUZZLE PROBLEMS

We will use our equation-solving skills to solve number problems, given visual and written clues.

Getting Started

This is a hundred chart.

1. State two patterns you notice on the chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Suppose a hundred chart is cut into puzzle-type pieces. Write numbers or algebraic expressions to represent values in each rectangle.

2.

		45	

3.

68	

4.

		28	

5.

n		

6.

	n

7.

n		

THE HUNDRED CHART PUZZLE

A hundred chart is cut up into 26 pieces. On each piece, one number is identified as *n*, and the sum (∑) of the numbers on the pieces is indicated. Follow the directions of your teacher. Use equation solving and logic to find the numbers on the pieces and to reassemble the chart.

PRACTICE 5

Let the variable n represent some number. Match the expression with the word descriptions. Some may be used more than once. Some may not be used at all.

_____	1.	5 more than a number	a.	$5n$
_____	2.	5 less than a number	b.	$\frac{n}{5}$
_____	3.	a number, increased by 5	c.	$n + 5$
_____	4.	The difference between a number and 5	d.	$\frac{5}{n}$
_____	5.	The product of a number and 5	e.	$n - 5$
_____	6.	The quotient of a number and 5	f.	$2(n + 5)$
_____	7.	Twice the sum of a number and 5	g.	$5 - n$
_____	8.	Twice a number, increased by 5	h.	$2n + 5$

9. Explain why $5 + n$ and $5n$ are not equivalent expressions.

Here are some puzzle pieces from a different hundred square puzzle. Use your equation-solving skills to find the numbers in the pieces.

10.

n		$\sum=138$

11.

	n	$\sum=92$

INTEGER PROBLEMS

1. Look up consecutive integers in section 2.5. Add the definition, one example using positive numbers, and one example using negative numbers to My Word Bank.

Your equation-solving skills can be useful for solving integer puzzle-type problems. Follow this outline to organize your work.

- Identify the variables.
- Write the equation.
- Solve the equation.
- Answer the question.
- Check your solution in the ORIGINAL problem.

2. The sum of three consecutive integers is 114. What is the middle integer?	3. One number is 8 more than another. Their sum is -42. Find the greater number.
4. The sum of 3 consecutive integers is -54. Find the middle integer.	5. The sum of 3 more than a number and twice the same number is 105. Find the number.

REVIEW

PERIMETER FORMULAS

1. A rectangle has perimeter *P*, length *L*, and width *W*.
   - Sketch the rectangle to the right.
   - Label the sides with *L* and *W*.

2. Write an equation for each description below.

   a. Giselle said, "I found the perimeter of the rectangle by adding length to width to length to width." Write the perimeter formula Giselle used.

   b. Marta said, "I found the perimeter of the rectangle by doubling the length, doubling the width, and then adding the two products." Write the perimeter formula Marta used.

   c. Alexandria said, "I found the perimeter of the rectangle by adding *L* and *W*, and then multiplying the sum by 2." Write the perimeter formula Alexandria used.

3. A rectangle has $L = 7$ cm and $W = 5$ cm. Use the formulas above to find the perimeter of the rectangle. Show all work.

a. Giselle's formula:	b. Marta's formula:	c. Alexandria's formula:

4. What do your answers in problem 3 suggest about the three formulas in problem 2?

TWO-STEP TARGET EQUATIONS

For problems 1-3, use this equation structure: $\square x + \square = \square$

1. Use exactly three of the digits 1 through 9 one time each. Write an equation and find its solution.

2. Use exactly three of the digits 1 through 9 one time each. Write an equation so that it has the greatest solution possible.

3. Use exactly three of the digits 1 through 9 one time each. Write an equation so that it has the least solution possible.

For problems 4-6, use this equation structure: $\square x - \square = \square$

4. Use exactly three of the digits 1 through 9 one time each. Write an equation and find its solution.

5. Use exactly three of the digits 1 through 9 one time each. Write an equation so that it has the greatest solution possible.

6. Use exactly three of the digits 1 through 9 one time each. Write an equation so that it has the least solution possible.

POSTER PROBLEM

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
The sum of four consecutive numbers is -118. Find the numbers.	One number is 5 more than four times another. Their sum is 75. Find the numbers.	The perimeter of a rectangular placemat is 50 inches. The length is 6 less than the width. Find the dimensions.	A triangular garden has side lengths that are consecutive numbers. It's perimeter is 54 ft. Find the side lengths.

A. Copy the problem. Draw a diagram (if possible) and identify the variable(s).

B. Write an equation to solve the problem. Do not solve it.

C. Solve the equation.

D. Answer the question. Check your answer in the original problem.

Part 3: Return to your seats. Work with your group, and show all work.

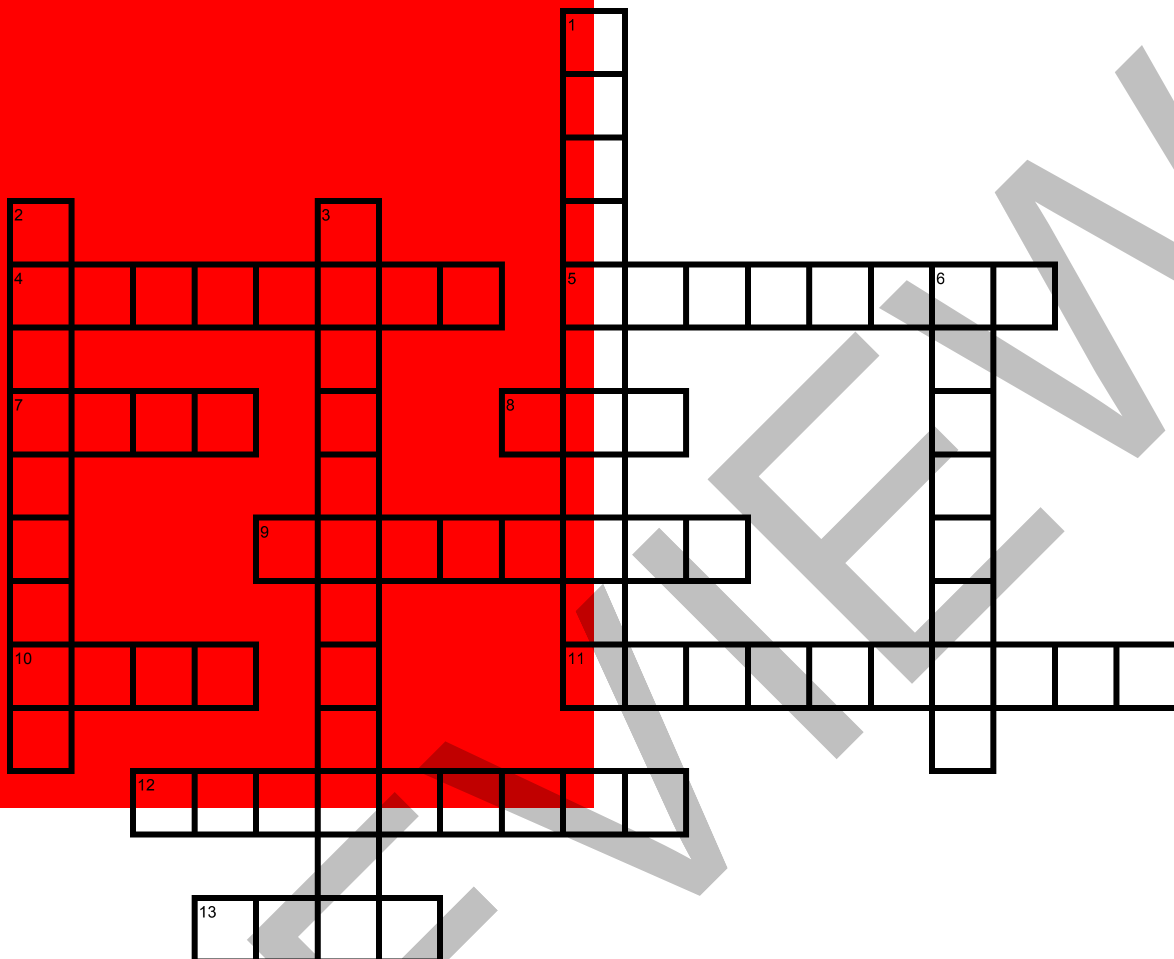
Critique your "start poster."

1. What mathematical ideas were displayed well on the poster?

2. What could be improved?

3. Rewrite the portion that could be better here.

VOCABULARY REVIEW



Across

4 A statement that asserts that two expressions are equal.

5 To find the value of an expression.

7 In the expression $3x + 7$, $3x$ is a ____.

8 The result of addition.

9 To write an expression in a more compact form.

10 Terms with the same variable part.

11 Two expressions that give the same value when evaluated for any value of the variable.

12 The distance around a figure.

13 The product of length and width in a rectangle.

Down

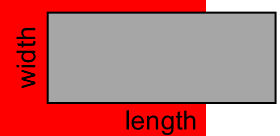
1 Integers such as 35, 36, 37, 38.

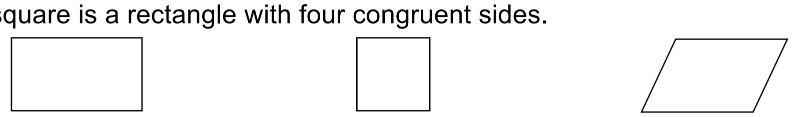
2 A polygon with 4 right angles.

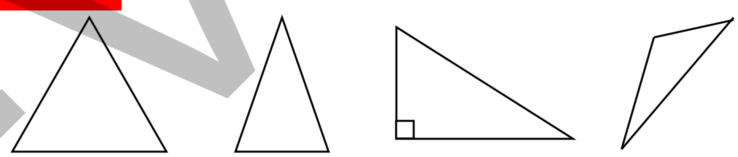
3 The property that rewrites products into sums.

6 A polygon with three sides.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
addition property of equality	The addition property of equality states that if $a = b$ and $c = d$, then $a + c = b + d$. In other words, equals added to equals are equal. If $3 = 2 + 1$ and $5(2) = 10$, then $3 + 5(2) = 2 + 1 + 10$.
area	The area of a two-dimensional figure is a measure of the size of the figure, expressed in square units. The area of a rectangle is the product of its length and its width.  Area = (length) × (width) If a rectangle has a length of 12 inches and a width of 5 inches, its area is $5 \times 12 = 60$ square inches.
coefficient	A coefficient is a number or constant factor in a term of an algebraic expression. In the expression $3x + 5$, 3 is the coefficient of the term $3x$, and 5 is the constant term coefficient.
consecutive integers	Two integers are consecutive if one of them is equal to the other plus 1. The integers 5 and 6 are consecutive, since $6 = 5 + 1$. The integers -20 and -19 are consecutive since $-19 = -20 + 1$. x, $x + 1$, and $x + 2$ are expressions for three consecutive integers
constant term	A constant term in an algebraic expression is a term that has a fixed numerical value. In the expression $5 + 12x – 7$, 5 and -7 are constant terms.
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. $3(4 + 5) = 3(4) + 3(5)$ and $(2 + 7)8 = 2(8) + 7(8)$
equation	An equation is a mathematical statement that asserts the equality of two expressions. $18 = 8 + 10$ is an equation that involves only numbers. $18 = x + 10$ is an equation that involves numbers and a variable. $y = x + 10$ is an equation that involves a number and two variables.

Word or Phrase	Definition
evaluate	Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression. To evaluate the expression $3 + 4(5)$, we calculate $3 + 4(5) = 3 + 20 = 23$. To evaluate the expression $2x + 5$ when $x = 10$, we calculate $2x + 5 = 2(10) + 5 = 20 + 5 = 25$.
equivalent expressions	Two mathematical expressions are equivalent if, for any possible substitution of values for the variables, the two resulting numbers are equal. In particular, two numerical expressions are equivalent if they represent the same number. See expression. The algebraic expressions $3(x + 4)$ and $3x + 12$ are equivalent. For any value of the variable x, the expressions represent the same number. The numerical expressions $3 + 2$ and $9 – 4$ are equivalent, since both are equal to 5.
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are $7x$, $a + b$, $4v – w$, and 19.
like terms	Terms of a mathematical expression that have the same variable part are referred to as like terms. See term. In the mathematical expression $2x + 6 + 3x + 5$, the terms $2x$ and $3x$ are like terms, and the terms 6 and 5 are like terms.
multiplication property of equality	The multiplication property of equality states that if $a = b$ and $c = d$, then $ac = bd$. In other words, equals multiplied by equals are equal. If $10 = 2 \bullet 5$ and $4 + 2 = 5 + 1$ then $10 \bullet (4 + 2) = 2 \bullet 5 \bullet (5 + 1)$ Check $60 = 60$
perimeter	The perimeter of a plane figure is the length of the boundary of the figure. The perimeter of a square is four times its side-length. The perimeter of a rectangle is twice the length plus twice the width. The perimeter of a circular disc is its circumference, which is π times its diameter.
rectangle	A rectangle is a quadrilateral with four right angles. In a rectangle, opposite sides are parallel and have equal length. A square is a rectangle with four congruent sides.  rectangle square not a rectangle

Word or Phrase	Definition
simplify	Simplify refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor. $2x + 6 + 5x + 3 = 7x + 9$ $\qquad$ $\frac{8}{12} = \frac{2}{3}$
term	A term in a mathematical expression involving addition (or subtraction) is a quantity being added (or subtracted). Terms that have the same variable part are referred to as like terms. The expression $2x + 6 + 3x + 5$ has four terms: $2x$, 6, $3x$, and 5. The terms $2x$ and $3x$ are like terms, since each is a constant multiple of x. The terms 6 and 5 are like terms, since each is a constant.
triangle	A triangle is a three-sided polygon. Triangles may be classified by their sides or their angles. If the three sides of the triangle have the same length, it is an equilateral triangle. If at least two sides have the same length, it is an isosceles triangle. If no two sides have the same length, it is a scalene triangle. If the three angles of a triangle have the same measure, it is an equiangular triangle. If all angles of the triangle are less than 90°, it is an acute triangle. If one of the angles of the triangle equals 90°, it is a right triangle. If one of the angles of the triangle is greater than 90°, it is an obtuse triangle.  classified by sides: equilateral, isosceles, scalene, scalene classified by angles: equiangular and acute, acute, right, obtuse
variable	A variable is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to inputs and outputs of functions, to quantities that vary in a relationship, or to unknown quantities in equations and inequalities. In the equation $d = rt$, the quantities d, r, and t are variables. In the equation $2x + 4 = 10$, the variable x may be referred to as the unknown.

Equivalent Expressions

Two numerical expressions are <u>equivalent</u> if they are equal.

$2 + 4$ and $-2 + 8$ are equivalent numerical expressions. They are both equal to 6.

Two mathematical expressions are <u>equivalent</u> if for any possible substitution of values for the variables, the two resulting values are equal.

The expressions $x + 2x$ and $4x - x$ are equivalent. For any value of the variable x, the expressions represent the same number. We see this by combining like terms.

$$x + 2x = 3x \text{ and } 4x - x = 3x.$$

The expressions x^2 and $2x$ are NOT equivalent. While they happen to be equal if $x = 0$ or $x = 2$, they are not equal for all possible values of x. For instance, if $x = 1$, then $x^2 = 1$ and $2x = 2$.

Properties of arithmetic, such as the distributive property, can be used to write expressions in different, equivalent ways.

$4x + 6x = (4 + 6)x$

$24x + 9x = 3(8x + 3x) = 3x(8 + 3)$

Simplifying Expressions Using a Model

In mathematics, we <u>simplify</u> a numerical or algebraic expression by rewriting it in a less complicated form.

We can illustrate simplifying expressions using a cups and counter model.

Positive Counter	Negative Counter	Cup
recorded as: **+** value: +1	recorded as: **–** value: -1	recorded as: **V** value: unknown (x)

Simplify $3(x + 2)$

Picture	Expression	What did you do?
V + + **V + +** **V + +**	$3(x+2)$ $= 3x+6$	Build the expression. (think: 3 groups of $x + 2$) Simplify. (the distributive property)

Simplify $2x + 3(x - 4)$

Picture	Expression	What did you do?
V V V – – – – **V – – – –** **V – – – –**	$2x+3(x-4)$ $= 2x+3x-12$ $= 5x-12$	Build the expression. (think: $2x$ plus 3 groups of $x - 4$) Picture shows the distributive property. Collect like terms to simplify.

Solving Equations Using Balance Strategies*

Illustrated here is one solution path for solving two examples using balance strategies. Others paths are possible to arrive at the same solutions.

As you solve equations, think:

→ Can I simplify each side? That is, focus on what can be done to each expression alone.

→ What can I do to both sides? That is, focus on what can be done to the equation.

Solve: $6 = 2(x + 1)$

Picture	Equation	What did you do?
+ + + │ V + + + + │ V +	$6 = 2(x + 1)$ or $6 = 2x + 2$	Build the equation. Rewrite using the distributive property.
+ + $\cancel{+}$ │ V $\cancel{+}$ + + $\cancel{+}$ │ V $\cancel{+}$	$6 = 2x + 2$ $-2 \quad\quad -2$ $4 = 2x$	Take away 2 from both sides.
+ + │ V + + │ V	$\frac{4}{2} = \frac{2x}{2}$ $2 = x$	Divide both sides by 2. To balance, each cup must contain +2

Check: $6 = 2(2 + 1) = 2(3) = 6$

Solve: $-3 = 3x + 6$

Picture	Equation	What did you do?
− − − │ V V V │ + + + │ + + +	$-3 = 3x + 6$	Build the equation.
− − − │ V V V − − − │ $\cancel{+}$ $\cancel{+}$ $\cancel{+}$ − − − │ $\cancel{+}$ $\cancel{+}$ $\cancel{+}$ │ $\cancel{-}$ $\cancel{-}$ $\cancel{-}$ │ $\cancel{-}$ $\cancel{-}$ $\cancel{-}$	$-3 = 3x + 6$ $+(-6) \quad\quad +(-6)$ $-9 = 3x$	Add -6 to each side. Remove zero pairs.
− − − │ V − − − │ V − − − │ V	$\frac{-9}{3} = \frac{3x}{3}$ $-3 = x$	Divide both sides by 3. To balance, each cup must contain -3.

Check: $-3 = 3(-3) + 6 = -9 + 6 = -3$

* These examples do not include the opposite of *x*. Equations involving (-*x*) will occur in the future.

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